Engineering Notes

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Exterior Ballistics of Free Fall Weapon Design

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Introduction

FOR the past several years there has existed a tripartite program directed toward examining, both in simulation and experiment, the dynamics of free-fall weapons. This program was a cooperative effort among laboratories in the United Kingdom, Australia, and the United States.[1] One particularly interesting aspect of the program was that it required a systematic examination of current theories of free-fall weapon stability.

Linear Exterior Ballistics

The formulation of the linear exterior ballistics of free-fall weapons is the product of many minds working over more than half a century. Two workers, Nicolaides[2] and Murphy,[3] deserve particular mention for their efforts in bringing linear theory to its present state of application. Their papers should be consulted for detailed derivations.

A free-fall weapon is acted upon by a force, $\bar{F}$, and a moment, $\bar{M}$. It can be shown that Newton's second law relates this applied load and load distribution to vectoral changes in linear and angular momenta. In component form the force and moment equations become

$$X=m[\dot{u}+qw-vr] \tag{1a}$$

$$Y=mV_o[\dot{\beta}+r] \tag{1b}$$

$$Z=mV_o[\dot{\alpha}-q] \tag{1c}$$

$$L=I_a\dot{p} \tag{1d}$$

$$M=I_T[\dot{q}+\upsilon pr] \tag{1e}$$

$$N=I_T[\dot{r}-\upsilon pq] \tag{1f}$$

where $\{X,Y,Z\}$ and $\{L,M,N\}$ are the components of force and moment along and about the body fixed $\{x,y,z\}$ axis frame. α and β are the angles of attack and sideslip while $\{p,q,r\}$ are the components of the angular velocity along the $\{x,y,z\}$ axes. I_T, I_a, and m are the transverse and axial moments of inertia and body mass.

Because a free-fall weapon usually has 90 deg rotational symmetry, it is possible to combine the force and moment equations as

$$Y+iZ=mV_o[\dot{\beta}+r+i\dot{\alpha}-iq]=mV_o[\dot{\xi}-i\bar{\Omega}] \tag{2a}$$

$$M+iN=I_T[\dot{q}+ir+\upsilon pr-i\upsilon qr]=I_T[\dot{\bar{\Omega}}-i\upsilon\bar{\Omega}p] \tag{2b}$$

where $\xi=i\alpha+\beta$ and $\bar{\Omega}=q+ir$. In coefficient form Eqs. (2) may be written as

$$\dot{\xi}-i\bar{\Omega}=\frac{QS}{mV_o}[C_y+iC_z] \tag{3a}$$

$$\dot{\bar{\Omega}}-i\upsilon\bar{\Omega}p=\frac{QSd}{I_T}[C_m+iC_n] \tag{3b}$$

A further consequence of 90 deg rotational symmetry is equality of parameters of yaw and pitch motion. For symmetric and asymmetric loads this equality may be expressed as

Symmetric Loads	Asymmetric Loads
$C_{Y_\beta}=C_{Z_\alpha}=C_{N_\alpha}$	$C_{y_o}=+C_{N_\delta}\delta\sin pt$
$C_{Y_{p\alpha}}=C_{Z_{p\beta}}=C_{N_{p\alpha}}$	$C_{Z_o}=-C_{N_0}=-C_{N_\delta}\delta\cos pt$
$C_{m_\alpha}=C_{n_\beta}=C_{M_\alpha}$	$C_{m_o}=xC_{N_\delta}\delta\cos pt$
$C_{m_q}=C_{n_r}=C_{M_q}$	$C_{n_o}=xC_{N_\delta}\sin pt$
$C_{m_{\dot\alpha}}=-C_{n_{\dot\beta}}=C_{M_{\dot\alpha}}$	
$C_{m_{p\alpha}}=C_{n_{p\alpha}}=C_{M_{p\alpha}}$	

where δ, for a statically stable body, is usually associated with fin cant. Using the above relationships in Eqs. (2) gives,

$$\xi+ip\bar{\Omega}=-(V/d)[C_N^*\bar{\xi}+iC^*_{N_{p\alpha}}(pd/2V)\bar{\xi}+iC^*_{N_\delta}\delta e^{ipt}] \tag{4a}$$

$$\dot{\bar{\Omega}}-i\upsilon p\bar{\Omega}=K_T^{-2}(V/2)^2\Big[-iC^*_{M_\alpha}\bar{\xi}+C^*_{M_q}\left(\frac{\Omega d}{2V}\right) -iC^*_{M_{\dot\alpha}}(\dot{\bar{\xi}}d/2V)+C^*_{M_{p\alpha}}(pd/2V)\bar{\xi}+ixC^*_{N_\delta}\delta e^{ipt}\Big] \tag{4b}$$

where the asterisk (*) indicates coefficients multiplied by the relative density term $\rho Sd/2m$, S being the reference area and d the reference length.

Equations (4) may be solved simultaneously while neglecting the product of starred terms, to give

$$\ddot{\bar{\xi}}+N_1\dot{\bar{\xi}}+N_2\bar{\xi}=N_3e^{ipt} \tag{5}$$

where

$$N_1\cong-(V/d)\{C^*_{N_\alpha}+K_T^{-2}(C^*_{M_q}+C^*_{M_{\dot\alpha}})+i(pd/2V)[K_T^{-2}C^*_{M_{p\alpha}}+C^*_{N_{p\alpha}}+2\upsilon]\} \tag{6a}$$

$$N_2\cong(V/d)^2\{-2K_T^{-2}C^*_{M_\alpha}+(pd/2V)[2i\upsilon C^*_{N_\alpha}-2iK_T^{-2}C^*_{M_{p\alpha}}]\} \tag{6b}$$

$$N_3\cong(V/d)^2\{2i(pd/2V)(1-\upsilon)C^*_{N_\delta}-2K_T^{-2}\tilde{x}C^*_{N_\delta}\}\delta_e \tag{6c}$$

Received Aug. 22, 1975; revision received April 5, 1976.

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Equation (5) may be solved to give:

$$\xi = K_n e^{(\lambda_n + i\omega_n)t} + K_p e^{(\lambda_p + i\omega_p)t} + K_t e^{ipt} \tag{7}$$

where

$$\lambda_{n,p} \cong (V/2d)\{C^*_{N_\alpha}(1 \mp \tau) + K_T^{-2}(C^*_{M_q} + C^*_{M_{\dot\alpha}})(1 \pm \tau) \pm 2K_T^{-2}/\upsilon\tau C^*_{M_{p\alpha}} \tag{8a}$$

$$\omega_{n,p} \cong (V/2d)\{2(pd/2V)\upsilon(1 \pm 1/\tau)\} \tag{8b}$$

$$K_t \cong \frac{(V/d)^2\{2i(pd/2V)(1-\upsilon)C^*_{N_\delta} - 2K_T^{-2}X_M C^*_{N_\delta}\}\delta}{[i(p-\omega_n) - \lambda_n][i(p-\omega_p) - \lambda_p]} \tag{8c}$$

$$\tau = \frac{1}{(1-1/s)^{1/2}} \qquad s = \frac{(pd/2V)^2(\upsilon^2)}{K_T^{-2}C^*_{M_\alpha}} \tag{9}$$

and where the subscripts n and p refer to nutational and precessional frequencies.

In Eq. (5) the independent variable is time t. It has been assumed that the coefficients N_1, N_2, and N_3 are constant and that the airspeed V and spin rate p are invariant during the integration process to arrive at Eq. (7). Since any free-fall weapon must evidence drag, ignoring velocity magnitude changes must be regarded as a simplifying assumption. Such an assumption is part of Nicolaides' work;[2] Murphy, on the other hand, includes drag and arrives at a constant-coefficient differential equation using arc length as the independent variable.[3]

Weapon Stability

By means of a simple diagram it is possible to illustrate the regions of stability and instability from which relevant design criteria may be established for a free-fall weapon with fixed-cruciform stabilizers of equal panel area. Figure 1 presents a plot of two of the most important parameters in free-fall weapon design, the reduced frequency $\hat{p}$ vs reduced pitch rate $\hat\omega_\alpha$.

Under conditions of steady spin, $\dot{p} = 0$, the reduced spin rate, $\hat{p} \equiv pd/2V$, may be written as

$$\hat{p} = C_{\ell_\delta}\delta / C_{\ell_p} \tag{10}$$

where C_{ℓ_δ}, C_{ℓ_p} and δ are the roll-driving moment derivative due to fin cant, the roll damping moment derivative, and the fin cant angle, respectively. The pitch frequency ω_α follows from Eq. (8b) with $p = 0$ and $\omega_p = \omega_n = \omega_\alpha$, as

$$\omega_\alpha^2 = -\frac{C_{m_\alpha}\rho V^2 Sd}{2K_T^2 md^2} \tag{11}$$

where K_T is the radius of gyration or $(I_T/md^2)^{1/2}$. Assuming the weapon to be a solid cylinder of fineness ratio f_R, it follows that

$$C_{m_\alpha} \cong -(4/\pi)\ell_T \bar{b}^2 C_{N_\alpha}/\mathcal{R}$$

$$m \cong \rho_b S d f_R$$

$$K_T^2 \cong f_R^2/12$$

where ρ_b is body density, $\bar{b}$ is the span of the tail surfaces, and ℓ_T the distance from the weapon c.g. to the aerodynamic center of the tail, both in calibers. Now defining the natural pitch frequency as $\hat\omega_\alpha \equiv \omega_\alpha d/2V$, Eq. (11) may be written as

$$\hat\omega_\alpha = [6/\pi)\ell_T \bar{b}^2 (C^T_{N_\alpha}/\mathcal{R})(\rho/\rho_b) f_R^{-3}]^{1/2} \tag{12}$$

It is of interest to look at upper bounds on both $\hat{p}$ and $\hat\omega_\alpha$. The reduced spin rate would obviously be limited by fin stall, an effect not accounted for in the formulation of Eq. (10). However, well within the bound of small fin-cant angles (less than 4 deg) the aerodynamic (Magnus) and yaw-pitch inertial coupling put a practical upper limit on $\hat{p}$.

Stability in nutation and precession depends upon

$$\lambda_{n,p} < 0 \tag{13}$$

Now Eq. (8a) can be simplified omitting the force term C_{N_α} and restricting attention to large values of spin rate $\hat{p}$, Eq. (9) thus becomes

$$\tau \cong 1 + (1/2s) = 1 - (2/\upsilon^2)(\hat\omega_\kappa/\hat{p})$$

where υ is the inertia ratio I_a/I_T and $C^*_{M_\alpha} = -4\hat\omega_\alpha^2 K_T^2$. Using the above expression, Eq. (8a) becomes in the limit as $\hat{p} \to \infty$,

$$\lambda_n = \left(\frac{V}{d}\right)\left\{K_T^{-2}(C^*_{m_q} + C^*_{M_{\dot\alpha}}) + \frac{K_T^{-2}}{\upsilon}C^*_{M_{p\alpha}}\right\} \tag{14a}$$

$$\lambda_p = \left(\frac{V}{d}\right)\left\{-\frac{2K_T^{-2}}{\upsilon}C^*_{M_{p\alpha}}\right\} \tag{14b}$$

According to Eq. (13), $\lambda_{n,p}$ must be negative. Since $C^*_{M_q} + C^*_{M_{\dot\alpha}}$ is always negative, stability depends upon the sign of the Magnus derivative $C^*_{M_{p\alpha}}$. If $C^*_{M_{p\alpha}}$ is negative, then nutational stability depends upon the relative magnitudes of $C^*_{M_{p\alpha}}/\upsilon$. Now $C^*_{M_q} + C^*_{M_{\dot\alpha}}$ is usually one to two orders of magnitude greater than $C^*_{M_{p\alpha}}$ but $1/\upsilon$ is between 10 and 30. Thus, it is possible to have a nutational instability for a positive Magnus moment. Clearly, for a negative Magnus moment, the vehicle is always unstable in precession. Thus at high spin rates an instability (Magnus) is likely. Further, according to Eqs. (14) this instability is independent of $C^*_{M_\alpha}$ and hence [through Eq. (11)] independent of $\hat\omega_\alpha$.

Returning to Eq. (12), it is clear that there is an upper bound to $\hat\omega_\alpha$ and one that is independent of $\hat{p}$. Clearly operational and stowage requirements severely limit tail length ℓ_T, body density ρ_b, and tail panel design $\bar{b}$, $C_{N_\alpha}^T$ and $\mathcal{R}$. Regions A and B in Fig. 1 represent forbidden spin and pitch frequency areas.

A further consideration in weapon dynamics is the avoidance of spin-yaw resonance. The possible occurrence of a resonance condition (a large increase in angle of attack) may be seen from Eq. (8c). Apparently the trim term K_T of Eq. (7) reaches a maximum when $|p - \omega_n|$ and $|p - \omega_p|$ are zero. The K_T term would be singular if no damping were present, i.e., if $\lambda_{n,p}$ were zero.

Defining $\hat\omega_{n,p}$ as $\omega_{n,p}(d/2V)$, it follows from Eqs. (8b) and (9) that

$$\hat\omega_n = (\hat{p}\upsilon/2)(1 \pm (1/\tau) = (\hat{p}\upsilon/2) \pm [(\hat{p}\upsilon/2)^2 + \hat\omega_\alpha^2]^{1/2} \tag{15}$$

where it has been recognized that $s = -(\hat{p}\upsilon/2)^2/\hat\omega_\alpha^2$. Since resonance is incipient when $\hat{p} \approx \hat\omega_\alpha$, and since $\upsilon = 0(0.1)$ and $\hat\omega_\alpha > p\upsilon/2$, it can be seen that $\hat\omega_p$ is of a sign opposite to $\hat\omega_p$. Thus $\hat{p}$ can only equal $\hat\omega_n$ and so resonance occurs when

$$\hat{p}/\hat\omega_\alpha = (1-\upsilon)^{-1/2} \approx 1 + \upsilon/2 \approx 1 \tag{16}$$

Thus region C in Fig. 1 isolates the resonance condition when the spin and pitch frequencies are equal or nearly equal.

Regions A, B, and C have been identified from arguments based upon linear analysis (low angles of attack). Region D, on the other hand, is associated with flight behavior where nonlinear aerodynamics predominate (high angles of attack).

Two aerodynamic phenomena characterize the motion of a fin-stabilized weapon in region D. One is concerned with the onset of fin stall, the other with aerodynamic cross-coupling effects caused by pitch-induced rolling moments and side moments (moments acting in a plane normal to the pitch plane). As the static stability of a weapon is reduced, the amplitude of its response to a specific disturbance tends to in-

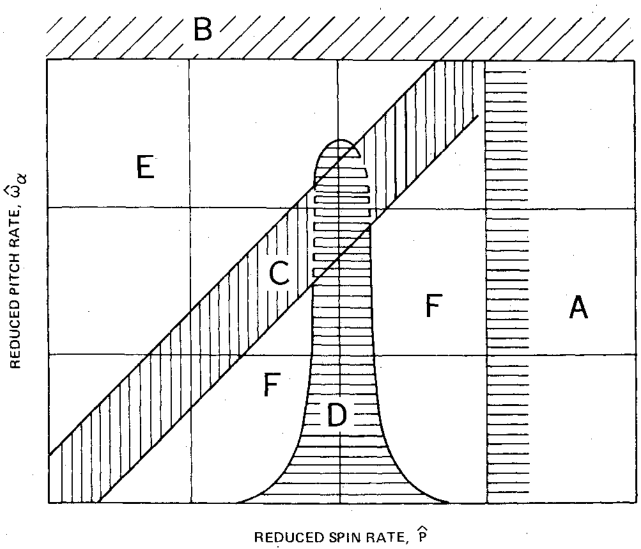

Fig. 1 Stability and design boundaries.

crease. This effect gives region D its "top-hat" appearance, with a wider region of instability occurring for the lower values of pitch rate. Clearly the shape and location of region D are not determined solely by weapon characteristics, but are significantly influenced by the nature and severity of the initial release disturbance. The so-called "catastrophic yaw search" is discussed at length in the summary report.[1]

Two regions of stable flight exist, shown in Fig. 1: a region E where the pitch rate is always higher than the roll rate and a region F where the converse is true. Again, the summary report[1] describes conditions at the lower left-hand corner of the diagram where regions E and F come together: fin alignments must be controlled to progressively smaller tolerances if the resonance region C is to be avoided.

References

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